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BASIC RESEARCH AND DATA ANALYSIS FOR THE NATIONAL GEODETIC SATELLITE PROGRAM AND FOR THE EARTH SURVEYS PROGRAM

Seventh Semiannual Status Report

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PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, OSU, and it is under the technical direction of Mr. Jerome, D. Rosenberg, Deputy Director, Communications Programs, OSSA, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C. 20546.

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1. STATEMENT OF WORK

The statement of work for this project includes data analysis and supporting research in connection with the following broad objectives:

(1) Provide a precise and accurate geometric description of the earth's surface.

(2) Provide a precise and accurate mathematical description of the earth's gravitational field.

(3) Determine time variations of the geometry of the ocean surface, the solid earth, the gravity field, and other geophysical parameters.

2. ACCOMPLISHMENTS DURING THE REPORT PERIOD

2.1 Experiments with SECOR Observations

2.11 Introduction

The experiments with the SECOR observations on GEOS-1, taken from the Pacific Tracking Network, have now been completed. During the previous reporting period results were given on two different solutions, referred to as SP-5 and SP-6. The final solution is referred to as SP-7, which is far superior to all earlier adjustments.

The SP-5 and SP-6 solutions gave a set of station coordinates that appeared to be reasonably consistent. However, there were certain quadrilaterals in the network that were weak due to an insufficient amount of data. It was suspected that much of the deleted data from earlier solutions was good, except that the observations contained constant biases. With the adjusted coordinates of the preliminary solutions (specifically the SP-5) it was now possible to perform short arc orbital mode adjustments for the purpose of recovering these biases.

2.12 Recovery of Ambiguity and Calibration Errors

The constant biases are made up of ambiguities, which occur in multiples of 256 meters, and calibration errors, which are generally under 30-40 meters. During the early stages of our experiments with the SECOR observations, a very extensive data screening procedure had to be established in order to select data that was free from ambiguities. This was very difficult, because at the time we did not have any ties between the observing stations.

The procedures used to arrive at the SP-5 and SP-6 solutions were described in the report for the last reporting period. To prepare for our SP-7 adjustment the final adjusted station coordinates from the SP-5 were used in short arc orbital mode adjustments to recover ambiguities from the data that was deleted earlier. This was accomplished by holding the station coordinates fixed and solving for ten parameters on each pass, the six elements that describe the orbit plus one range error term for each observing station. By performing several iterations on each pass, we were able to remove ambiguities from 14 additional passes of data. Since ambiguities are multiples of 256 meters, we were able to constrain these on each of the 14 passes and solve for the calibration error also.

With the addition of the 14 new passes, this m. de a total of 67 passes of data that were free from ambiguities. The time spans of all the data used are listed in Table 1. Passes 1-47 were the data selected during the data screening phase of the experiments, passes 48-53 were the passes of EGRS-7 data received from the U. S. Army Topographic Command, and passes 54-67 were the additional 14 passes of data that are now free from ambiguities.

It was also possible to make a reasonable estimate of the calibration errors for some of the data that constituted only a very short segment of an arc. In many instances it was noted that residuals for a given station in the geometric mode solution were fairly large, constant, and of the same sign. For these observations, the mean residuals served as estimates of the calibration error.

Table 2 is a listing of the ambiguity and calibration corrections that were determined. This data was removed, the corrections applied, and then added back into the usable set of data.

2.13 The SP-7 Solution

The SP-5 and SP-6 solutions were based on an origin established by observations made from the Coast and Geodetic Survey's Worldwide Geometric Satellite Network (BC-4). The coordinates at Johnston Island was determined from the observations of a PC-1000 camera that co-observed with three BC-4 cameras. These station coordinates were referenced to the North American Datum. For the SP-7 solution the geocentric coordinates of the Maui Baher-Nunn station defined the origin, their weights based on the standard derivation of seven meters for each Cartesian component as given by SAO. This was possible because of Maui there was also a camera station from the C & GS Worldwide Network, and both stations were tied into the local survey system together with

Table 1. Timespans of Data Used

Satellite	Pass No.	Date 1966		From h	From m	From s	To h	To m	To s
GEOS - I	1	6	4	7	55	12			
GEOS - I	2	6	4	18	43	28	18	50	40
GEOS - I	3	6	5	18	50	36	18	50	40
GEOS - I	4	6	8	6	00	20	6	02	56
GEOS - I	5	6	17	15	30	04	15	33	16
GEOS - I	6	6	18	4	38	56	4	42	48
GEOS - I	7	6	19	4	44	04	4	46	44
GEOS - I	8	6	20	4	46	24	4	54	24
GEOS - I	9	6	26	14	08	00	14	09	44
GEOS - I	10	6	28	3	12	08	3	19	52
GEOS - I	11	7	3	1	31	40	1	37	24
GEOS - I	12	7	5	1	40	32	1	48	28
GEOS - I	13	7	7	23	53	16	23	58	32
GEOS - I	14	7	9	10	51	12	10	51	16
GEOS - I	15	7	9	21	59	28			
GEOS - I	16	7	10	0	01	28	00	09	48
GEOS - I	17	7	11	11	02	16			
GEOS - I	18	7	12	11	03	20			
GEOS - I	19	7	14	22	22	32	22	22	36
GEOS - I	20	7	15	22	19	16	22	29	48
GEOS - I	21	7	16	9	17	44	9	20	20
GEOS - I	22	7	16	22	24	00	22	29	16
GEOS - I	23	7	17	9	20	08	9	25	28
GEOS - I	24	7	17	22	30	04	22	35	24
GEOS - I	25	7	19	4	44	04	4	46	44
GEOS - I	26	7	21	20	43	24	20	48	44
GEOS - I	27	7	22	20	44	24	20	52	20
GEOS - I	28	7	23	7	40	44	7	40	48
GEOS - I	29	7	23	18	47	00			
GEOS - I	30	7	23	20	56	56	20	59	36
GEOS - I	31	7	24	7	46	44	7	49	24
GEOS - I	32	7	24	18	50	24	18	53	04
GEOS - I	33	7	24	20	58	44	21	05	28
GEOS - I	34	7	25	18	55	24			
GEOS - I	35	7	26	18	55	24	19	02	08
GEOS - I	36	8	6	17	40	48	17	46	32
GEOS - I	37	11	5	14	57	24	15	00	00
GEOS - I	38	11	5	21	06	08	21	12	12
GEOS - I	39	11	17	17	48	04	17	52	00
GEOS - I	40	11	19	17	57	20	18	01	24
GEOS - I	41	11	20	18	05	16	18	10	32
GEOS - I	42	11	22	18	10	56	18	16	12
GEOS - I	43	11	23	16	14	44			
GEOS - I	44	11	24	16	19	28	16	22	08
GEOS - I	45	11	25	16	22	24			
GEOS - I	46	11	26	16	28	04	16	33	20
GEOS - I	47	11	27	16	31	20	16	36	36
EGRS-7	48	12	18	16	07	40	16	12	40
EGRS-7	49	12	20	15	39	03	15	45	43
EGRS-7	50	12	21	14	05	47	14	06	27
EGRS-7	51	12	22	15	12	22	15	16	37
EGRS-7	52	12	24	14	39	00	14	43	45
EGRS-7	53	12	27	15	21	16	15	25	00
GEOS - I	54	5	24	22	04	07	22	07	07
GEOS - I	55	6	7	18	56	44	18	59	48
GEOS - I	56	6	11	6	13	48	6	16	04
GEOS - I	57	7	21	9	34	44	9	37	20
GEOS - I	58	7	27	19	01	56	19	05	44
GEOS - I	59	8	22	14	39	56	14	46	44
GEOS - I	60	9	5	11	30	12	11	37	44
GEOS - I	61	9	23	6	34	40	6	36	24
GEOS - I	62	10	20	0	09	12	0	10	20
GEOS - I	63	12	6	15	03	52	15	09	48
GEOS - I	64	12	9	13	10	04	13	15	04
GEOS - I	65	12	11	13	19	08	13	23	04
GEOS - I	66	12	16	11	39	36	11	41	48
GEOS - I	67	* 1	8	6	55	44	7	03	36

* 1967

Table 2. Recovered Ambiguity and Calibration Corrections

(In Meters)

Station	5401		5402		5403		5404		5405		5406		5407		5408		5410		5411	
Pass No.	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal
1				-3		+3				-2		+3								
3				+3		-2				+2		-3								
5				+5		-6				+4		-4								
6		-5		-5		+4		+5												
10		+5		+6		-5		-6												
11		+2		+3		-2		-3												
16				-5		+6				-5		+5								
16a				+5		-6				+4		-4								
20				+2		-2				+2		-3								
20a				+6		-8				+6		-3								
30				-5		+7				-5		+3								
35								+5		-5		-3		+4						
38						+1						-3		+4		-3				
46						+1						-2		+3		-3				
49						-2										+6		0		-5
50						-8										+15		-1		-8
51						-5										-15		0		+12
53						-3										+13		0		-12
54	-256		-512		-256		-512													
55			-256	+1	0	-1			0	+1	-256	-2								
55a			-256	+3	0	-2			0	+2	-256	-4								
56			-256	-2	-256	+1			0	-1	-256	+2								
57			-512		0				-256		-512									
58							256	-6	256	+6	-256	+5	-256	-6						
58a							256		256		-256		-256							
59							-256		-512		-768		-512							
60							-512	+3	-768	-5	-768	-3	-768	+6						
61					-256	-1					0	+2	0	-3	0	+2				
62					-768						-256		-256		-768					
63					-1280	0					-768	0	-768	+1	-768	-1				
64					-768								-256				-512		-256	
65					-768								-512				-512		-512	
66					-768								-256				-512		-512	
67					512	+2									256	-8	0	-1	0	+9
67a					512	-1									256	+3	0	0	0	-8

the Maui SECOR station. The orientation was aided by constraining the directions from Johnston to Maui and Johnston to Midway, the same orientation as in the SP-5 solution. A change from the previous solutions was that the geodetic heights of all SECOR stations were constrained, and they were constrained with weights corresponding to a standard deviation of 15 meters. The adjustment was made on the SAO ellipsoid with dimensions a = 6378155 and f = 1/298.255. There were a total of 1188 range observations (at $4^s - 60^s$ intervals) which, with the external constraint equations, resulted in 287 degrees of freedom. Tables 3 and 4 give the relative positions from local ground surveys, and the station coordinates used in the network orientation.

Table 3. Relative Positions from Local Ground Surveys

From To	Name	Type	Δx(m)	Δy(m)	Δz(m)	Estimated σ in each coordinate (m)
5408 3475	Johnston Johnston	SECOR PC-1000	3.8	0.8	-1.2	0.5
5411 6011	Maui Maui	SECOR BC-4	2001.2	-22992.3	-10965.0	0.5
5411 9012	Maui Maui	SECOR BN	1951.7	-22873.4	-11000.9	0.5
6011 9012	Maui Maui	BC-4 BN	-49.5	118.9	-35.9	0.5
5410 2724	Midway Midway	SECOR Doppler	-882.6	1911.2	-1481.4	0.5

2.14 Results

The adjusted coordinates from all three solutions are shown in Table 5. For ease of comparison, the coordinates of the SP-5 and SP-6 solutions have also been converted to the SAO-1969 system to be compatible with the SP-7 solution. In the solutions SP-5 and SP-6, the standard deviation of a single range estimated a posteriori from the solution was 8.6 meters. The SP-7 solution

Table 4. Station Coordinates Used in the Network Orientations.

GOCC#	Name	Type	Datum	Latitude	σ	Longitude (+E)	σ	h (m)	σ(m)	Note
6011	Maui	BC-4	NAD	20°42′26.″139	0.″351	203°44′42.″886	0.″396	3001.4	12.0	1
9012	Maui	Baker-Nunn	SAO 1969	20 42 25.66	0.250	203 44 33.48	0.25	3029.0	7.0	5
6012	Wake	BC-4	NAD	19 17 28.247	0.470	166 36 43.564	0.515	-159.2	17.2	1
6059	Christmas	BC-4	NAD	2 0 13.185	0.487	202 35 20.508	0.380	-22.4	13.3	1
3475	Johnston	PC-1000	NAD	16 43 44.209	0.254	190 28 49.931	0.313	-90.2	7.9	2
2724	Midway	Doppler	Mercury	28 11 48.79		182 36 40.13		-14		3
2724	Midway	Doppler	NAD	28 11 50.47		182 36 46.16		-117		4

Notes:

1. Coast and Geodetic Survey preliminary coordinate.
2. Obtained at OSU by adjusting the ACIC optical data, weighting the coordinates of Maui, Wake, and Christmas according to their uncertainties.
3. NWL-8D Solution [Anderle and Smith, 1967]. Uncertainty is 25 m in each Cartesian coordinate.
4. Obtained from NWL-8D Mercury Datum Coordinate, using translation parameters of $\Delta x = -40$ m, $\Delta y = 163$ m, $\Delta z = 186$ m [Anderle and Smith, 1967].
5. Computed from Cartesian coordinates, with $\sigma = 7$ m for each coordinate, and referred to the SAO ellipsoid of the following parameters: $a = 6378155$ m, $1/f = 298.255$ [Gaposchkin and Lambeck, 1969].

reduced this standard deviation to 3.2 meters. As can be seen by examining Table 5, the additional data, and the removal of the systematic errors from the existing data, made the SP-7 solution far superior to any of the earlier adjustments.

Table 6 gives the geodetic coordinates of the SP-7 solution on the North American Datum. To transfer the coordinates from the SAO system to the NAD, the following translation parameters were used:

$$\Delta x = 38\,\mathrm{m}, \quad \Delta y = -164\,\mathrm{m}, \quad \Delta z = -175\,\mathrm{m}.$$

These parameters are in the sense NAD-SAO.

2.15 Conclusions

Our experiences with the SECOR observations of GEOS-I in the Pacific indicate that with a great deal of effort one can obtain satisfactory solutions. Since none of the observing stations are positioned on major datum, external information must be used to tie the network into existing coordinate systems. Since ambiguity and calibration corrections can be extracted reliably only from those data subsets that constitute passes, and only a very few of the passes are long enough to allow the use of an error model more extensive than the single constant bias term, small systematic errors are still suspected to be present in some of the data.

The solutions for the station coordinates (Table 5 and 6) appear to be completely valid. The standard deviations of the coordinates are all acceptable. There seems to be some rise in the standard deviations toward the western and southern parts of the network, probably because all direction control is in the northeastern part of the net. If ballistic camera data or other directional information were available from some of the stations on the western end, the whole network could be further strengthened.

Table 5. SAO - 1969 Coordinates

GOCC #	NAME		SP-5	σ	SP-6	σ	SP-7	σ
5401	Truk	x	-5576046 m	20 m	-5576050 m	20 m	-5576050 m	12 m
		y	2984663	22	2984651	24	2984667	12
		z	822370	35	822391	41	822438	15
5402	Swallo	x	-6097439	14	-6097445	14	-6097450	8
		y	1486476	27	1486472	33	1486518	15
		z	-1133253	23	-1133237	27	-1133244	10
5403	Kusaie	x	-6074526	13	-6074532	14	-6074527	8
		y	1854349	17	1854340	20	1854359	10
		z	583794	23	583811	28	583838	11
5404	Gizo	x	-5805386	16	-5805390	16	-5805394	9
		y	2485301	27	2485295	32	2485342	14
		z	- 892928	29	- 892947	35	- 892882	12
5405	Tarawa	x	-6327917	11	-6327924	13	-6327924	7
		y	784564	18	784558	22	784583	11
		z	150802	17	150815	20	150834	9
5406	Nandis	x	-6070188	19	-6070195	19	-6070207	11
		y	270635	35	270636	41	270690	18
		z	-1932863	21	-1932851	23	-1932851	11
5407	Canton	x	-6304300	16	-6304305	16	-6304308	9
		y	- 917656	22	- 917657	26	- 917626	13
		z	- 307105	15	- 307097	15	- 307106	9
5408	Johnston	x	-6007969	8	-6007974	6	-6007981	5
		y	-1111233	9	-1111238	8	-1111240	8
		z	1824153	8	1824160	7	1824156	7
5410	Midway	x	-5618708	15	-5616715	13	-5618721	10
		y	- 258181	20	- 258193	13	- 258217	10
		z	2997221	21	2997228	13	2997241	10
5411	Maui	x	-5468005	11	-5468005	9	-5468010	6
		y	-2381408	12	-2381408	9	-2381410	7
		z	2253172	8	2253172	10	2253175	7

Table 6. North American Datum Coordinates

(Solution SP-7)

GOCC #	Name	Latitude	σ	Longitude (+E)	σ	Height	σ
5401	Truk	7°27′27.″1	0.″5	151°50′33.″2	0.″4	-127 m	12 m
5402	Swallo	-10 18 18.7	0.3	166 18 01.1	0.5	- 39	8
5403	Kusaie	5 17 10.4	0.3	163 01 32.0	0.4	- 84	7
5404	Gizo	- 8 06 12.7	0.4	156 49 30.1	0.5	- 21	10
5405	Tarawa	1 21 45.9	0.3	172 56 0.7	0.4	- 95	6
5406	Nandis	-17 45 36.8	0.3	177 26 53.6	0.6	53	11
5407	Canton	- 2 46 48.8	0.3	188 16 59.0	0.4	- 23	9
5408	Johnston	16 43 44.0	0.2	190 28 50.2	0.3	-105	5
5410	Midway	28 12 45.4	0.3	182 37 58.6	0.4	-121	10
5411	Maui	20 49 24.6	0.2	203 32 7.9	0.2	- 24	6

2.2 Satellite to Satellite Tracking

This investigation was completed during the reporting period. A detailed description of this investigation is being published separately as Reports of the Department of Geodetic Science No. 149, "Gravity Field Refinement by Satellite to Satellite Doppler Tracking."

The main objective of this investigation was to find what resolution of the gravity field may be obtained from satellite to satellite Doppler tracking. This question was answered by performing least squares adjustments of simulated satellite to satellite Doppler data, solving for parameters describing the anomalous gravity field. These parameters were the mean values in various size blocks of the density of a fictitious surface layer, although mean gravity anomalies could have been used just as well. By examining the correlation between the adjusted parameters describing neighboring blocks, it was possible to judge whether a given set of data was capable of resolving blocks of a given size.

Two concepts of satellite to satellite Doppler tracking were considered. The first concept uses the range rate between two satellites near together in very low orbits. In the second concept, a constellation of very high geostationary satellites track a single satellite in a very low orbit. In both cases, the obtainable resolution was found to depend directly on the altitude of the low satellite. From an altitude of 700 km, blocks 500 km on a side were satisfactorily resolved. Blocks 200 km on a side may be satisfactorily resolved from an orbital altitude of 200 km. This altitude is about the lowest at which a satellite can be kept in orbit for the length of time necessary to survey the entire earth, even with a drag compensation device. Because of the lower limit on altitude imposed by the presence of the earth's atmosphere, it does not appear that satellite to satellite Doppler tracking will be able to resolve features smaller than 200 km on a side.

Although both concepts of the configuration of the two satellites yielded satisfactory results, the resolution was slightly better when two satellites near together in very low orbits were used. On the other hand, a set of very high geostationary satellites which track a single low satellite may also be used for several other purposes, so that this latter concept is recommended over the concept of two very low satellites.

In addition to the altitude of the lower satellite, several other design parameters were varied to determine their effect on the resolution of the gravity field. It was found that some ground tracking of the low satellite is necessary in order to provide some geographic location to the anomalous gravity whose effect is being observed. However, unless very long orbital arcs are tracked, all of the gravimetric information will be contained in the satellite to satellite range rate, so that precise tracking of satellite position from the ground will be neither necessary nor desirable.

The data rate and spacing between ground tracks should be sufficient to insure that at least one, and preferably more, measurements of satellite to satellite range rate are made with the low satellite over each block. This means that for a satellite at an altitude of 200 km, and the gravity field represented in 200 km blocks, a data rate of one range rate measurement every 30 seconds would be just sufficient.

Some variation in the relative geometry of the two satellites was found to be desirable. When two satellites in low orbits are used, the solution is strengthened if the two satellites are one behind the other in the same orbit on some passes, one above the other in the same orbit plane on other passes, and roughly side by side in slightly different orbit planes in still other passes. If a single low satellite is tracked by geostationary satellites, variation in the relative geometry of the two satellites is more difficult to achieve, so that it may be desirable to use several low satellites at different inclinations. However, satisfactory results can be obtained with a single low satellite, so that the use of several low satellites at different inclinations is not essential.

2.3 Investigations Related to Range Observations

During the second half of 1971 many computer runs were performed in connection with analysis of critical configurations in the range observational mode. Also further theoretical investigations were conducted with respect to critical configurations when the concept of "leapfrogging" is applied to range observations. The ground stations were assumed to be lying in a plane, which is approximately fulfilled in practice when the network extends over a relatively small territory. The coordinate systems used in numerical computations of all the investigated cases were always chosen in such a way as defined by the "inner adjustment constraints" as described below. A considerable amount of time was devoted to the complete theoretical treatment of such constraints and their detailed description is being published separately as Reports of the Department of Geodetic Science No. 148, "Inner Adjustment Constraints with Emphasis on Range Observations".

Adjustments of geodetic networks are commonly made in terms of the actual coordinates of points in some coordinate system. Computer programs which carry out solutions for such networks operate with coordinates. To use these programs, a coordinate system for the adjustment must be defined in some manner.

One kind of geometric observation cannot provide all the necessary information about the coordinate system (origin and orientation), and the scale, which are needed for computation of the coordinates of points involved in an adjustment. For instance, optical observations of satellites alone offer no information about the scale or the origin of a coordinate system. Range observations of satellites alone can provide no information about a coordinate system, of which they are independent, while the scale is inherent in the observations. In this latter case the number of elements to be specified in order to define a coordinate system is six: Three parameters to define its origin and three parameters to define its orientation. This could be effectively done by selecting six coordinates (distributed over more than two points) in the network to be held fixed in an adjustment.

When only six constraints are used in the adjustment of range observations to define a coordinate system, they are called a minimal set of constraints. This expresses the fact that a minimum of six constraints are needed to obtain a unique solution in terms of coordinates. When the set of constraints is more than minimal then the adjustment is called over-constrained. This happens in practice when more real information about the network is available than that represented by six constraints.

Often in practice, it is very important to analyse the data (ranges) in some network or part thereof in order to detect suspicious observations. This can be accomplished by using any set of minimal constraints. Since these constraints define uniquely a coordinate system of which the observations are invariant, theoretically the same adjusted observations and therefore the same set of residuals should be obtained from an adjustment, no matter which particular minimal set of constraints was used, i.e. no matter how the coordinate system was defined.

Incorporating the six minimal constraints into the adjustment of a network can be easily accomplished by the proper choice of six coordinates to be held fixed. Even though the residuals are uniquely defined for any such choice, certain numerical difficulties may arise due to a weak definition of the coordinate system. Namely, if the coordinates to be held fixed are not selected with care, the poor definition of the coodinate system is reflected in poor propagation characteristics, resulting in some excessively large numbers in the inverse matrix of normal equations. It is therefore advisable to define the coordinate system for data analysis the best possible way, thus bringing the inherent numerical difficulties to a minimum. In this sense "best" is interpreted as resulting in the smallest trace of inverse matrix of normal equations, or equivalently, the smallest trace of the variance-covariance matrix for the parameters (coordinates). The minimal constraints which materialize this "best" coordinate system are called inner adjustment constraints.

Sometimes it may occur that (range) observations are performed in an isolated area, where none of the points forming the network is known on any datum. In such a case, if an adjustment is to be performed, a coordinate system (datum) can be defined arbitrarily using a minimal set of constraints. It would be of natural interest to define it in such a way that after the adjustment the mean point error of the points of interest would be the smallest possible or equivalently to require that the trace of the variance-covariance matrix for these points after an adjustment be minimum compared to all other coordinate systems defined differently. According to the above discussion such a "best" coordinate system can be arrived at through the use of the inner adjustment constraints for the points of interest.

Another area where use of the inner adjustment constraints is advantageous is the analysis of critical configurations for the range observational mode (or analogously for other observational modes with proper modifications). If the configurations of points in a network leading to singular or nearly singular solutions are to be detected, it is important that numerical problems due to the weak definition of the coordinate system be eliminated. This was the purpose of using the inner adjustment constraints in the past investigations.

For the range observational mode, the (6 x u) matrix C, representing inner adjustment constraints, has the form:

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & 1 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & 1 & \\ 0 & \bar{z}_1 & -\bar{y}_1 & 0 & \bar{z}_2 & -\bar{y}_2 & \\ -\bar{z}_1 & 0 & \bar{x}_1 & -\bar{z}_2 & 0 & \bar{x}_2 & \cdots \\ \bar{y}_1 & -\bar{x}_1 & 0 & \bar{y}_2 & -\bar{x}_2 & 0 & \end{array}\right].$$

The coordinates $\bar{x}, \bar{y}, \bar{z}$ are approximate coordinates of the points of interest, scaled by a constant k (e.g. $k = 1/206,265$). In practice these points often correspond to the ground stations, yielding thus after an adjustment:

$$Tr(\Sigma gr) = \text{minimum},$$

where Σgr denotes the variance-covariance matrix for the ground stations.

Investigations dealing with the critical configurations of points when ranges are used will be soon completed and published as a separate report.

2.4 Investigations Related to the Problem of Improving Existing Triangulation Systems by Means of Satellite Super-Control Points

Different adjustment methods were studied with a view to find a very suitable method which would not only adjust a large geodetic triangulation system simultaneously i.e. to use the entire data as a whole, but also to use the original observation equation matrix (A-Matrix) directly thus avoiding the formation of normal equation, where certain useful characteristics of A-Matrix, such as very small coefficients may be lost. As such the Method of Conjugate-Gradients (Cg-Method) was selected; it is an iterative method, developed by Stiefel [Stiefel 1952; Wolf, 1968], gives the solution vector theoretically after n-iterations, where n = number of unknowns. Worth mentioning here are two advantages of Cg-Method over direct methods, where normal equations are formed:

(1) Original A-Matrix of a triangulation system which has very few non-zero elements, is easily stored in a comparatively much less computer space, using an Index Matrix (I-Matrix), rather than N-Matrix. Thus the same computer can solve larger systems simultaneously if A-Matrix is used rather than N-Matrix.

(2) No "mesh-point numbering technique" [Ashkenazi, 1967] to keep the bandwidth of the system a minimum is necessary. Thus stations can be added or taken out from the existing triangulation system without caring for their numbers.

Advantages of Cg-Method over other iterative methods, such as Gauss-Seidel, Jacobi, Method of Steepest Descent are mainly a good convergence, giving the solution vector in finite iteration steps, and use of original A-Matrix [Wolf, 1968; Stiefel, 1952; Hilger et al, 1967].

After selecting Cg-Method for adjustment of large triangulation systems, this was programmed, which is valid (at present) for 1000 unkowns and 2000 observation equations. This program has been used for smaller examples up to 39 unknowns and 66 observation equations, and was found most satisfactory. This program will now be used for the triangulation chain between Moses Lake and Chandler, the data of which was received by USCGS. This triangulation chain contains 273 stations (5 stations are omitted), 37 bases (out of which 5 Geodimeter Bases), 18 Laplace stations and 1340 observed directions. The total chain-length is ~ 1858 km. This data has been processed here to form homogenized A-Matrix of (1395, 819) dimension. The standard deviations of the observed data as recommended by Mr. Meade [Meade's letters dated July 13 and July 30, 1970] were used to homogenize the A-Matrix.

Although theoretically n-iterations give the exact solution, it is not true for large systems due to the round-off errors of computational procedure. The number of iterations, which should give an exact solution vector, depends upon the condition of A-Matrix and conjugacy of residue vectors. It is difficult to mention at this stage how many iterations a particular system will require to render the solution vector as till now published reports, mostly by mathematicians, show varying numbers — n to 3/2 n iterations [Hilger et al, 1967], up to 3 n steps [Ginsburg, 1963] etc. Even badly conditioned systems are solvable by Cg-Method, naturally with more iterations.

Covariance Matrix Program using Cg-Method has been developed and is under test.

Data for 98th meridian arc have been recently received from USCGS, which will be processed in the next reporting period.

References

Ashkenazi, V. (1967). "Solution and Error Analysis of Large Geodetic Networkes", Survey Review, No. 146.

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Hilger, F. and Remmer, W. (1967). Das Rechenprogramm AUGL zur netzweisen Ausgleichung einzelner oder mehrerer Triangulations ordnungen in einem Guss, DGK, Reike B/139.

Stiefel, Edward (1952). "Über einige Methoden der Relaxationsrechnung", ZAMP, III, Basel.

Wolf, Helmut (1968). Ausgleichungsrechnung n. d. M. d. k. Q., Dümmlers Verlag, Bonn.

2.5 Investigations Related to C-Band Radar Observations

On September 10, 1970, Jim Reilly and Marvin Whiting visited the offices of Wolf Research and Development Company in Pocomoke City, Maryland meeting Ron Brooks, manager of the C-Band data analysis for Wolf. They then accompanied Brooks to the Wallops Island Station to meet Ray Stanley, NASA's Project Manager for the C-Band System Project.

Here they were able to ask a number of questions about the C-Band System which were answered with great clarity by Ray Stanley. The methods of synchronizing the time systems of the various C-Band stations through data analysis, the location of the surveyed station at each C-Band installation, the differences of the differently named radar types used in the system, the resolution attained by the C-Band relative to the lasers and the nature of the time recorded, were all made clear. Mr. Stanley volunteered to obtain the needed ties between the C-Band on Johnson Island and the Baker Nunn camera located there.

The next day, Ray Stanley conducted a tour for Reilly and Whiting of the Wallops Island installation, most particularly showing them the FPQ-6 and FPS-16 radars. While visiting the latter, a weather rocket was launched and they were able to observe the radar while it was tracking.

Reilly and Whiting again visited Ron Brooks and obtained a list of those biases in the radar's time and range measurements which Wolf Research had been able to discover during its analysis of the data over the last two years. He also explained other station peculiarities which might affect the C-Band data.

TIMING CORRECTIONS NECESSARY

WOOMERA	4860	+25 m sec.
VANDENBERG AFB	4280	-22 m sec.
WALLOPS ISLAND	4840 & 4860	January 28, 1969 to January 30 0^h -100 m sec.
		January 30 0^h to February 5 12^h -200 m sec.
		February 5 12^h to February 13 12^h -100 m sec.

(Advised not to use Wallops Island data after February 13 12^h)

MAKAHA RIDGE	4402	+10 m sec.
		February 9 0^h to February 13 0^h +1.01 sec.

RANGE CORRECTIONS NECESSARY

KAVAI H.I.	4742	+52 meters
BERMUDA FPS-16	4740	- 3 meters
BERMUDA FPQ-6	4760	-18 meters
ANTIQUA	4061	-17 meters
GRAND TURK	4081	-18 meters
CANAVON	4761	-12 meters

WOOMERA is a 500 mile radar and will have 30 sec. tracking gaps

Adivsed not to use Canary Island 4720 (noisy radar MPS-26), Tananarive (questionable calibration procedure), and Canavon (timing error in every increment I, error = nI with n unknown, also G. P. uncertain).

After returning from Wallops Island we have been engaged in a number of unrelated projects. Firstly the modifications of the C-Band data suggested by the visit to Wallops Island were carried out. Using only data from a selected number of stations in the United States and Bahamas we used the OSU short arc adjustment program to solve for station position for four passes. This was to familiarize ourselves with the behavior of the data.

This work revealed that the orbit model of this program was inadequate for the longer passes and more accurate observations of the C-Band System. The decision was made to improve the orbit model before continuing further with the C-Band project.

Related to the C-Band project a program was written to convert from geodetic coordinates to a geocentric system and all the station positions of the C-Band radar were converted to geocentric SAO 1969 position. The intention was to use them as preliminary values for the adjustments to come.

Next it became necessary to update the polar motions tables contained in several programs. This was simple enough for the programs that run on the 7094 but more complex for the 360, since the programs are permanently stored on a disk, and some study of the IBM manuals was necessary before the updating could be attempted.

2.6 Computer Programming Efforts

As mentioned in the previous section it has become apparent that the short arc orbital mode adjustment computer programs which we have been using are not always adequate for our needs. The computer programs available at The Ohio State University for performing network adjustments of satellite observations in the short arc mode are (1) the program written at OSU and briefly described in the First Semi-Annual Status Report for this project, and (2) the Short Arc Geodetic Adjustment (SAGA) program, written for AFCRL by Duane Brown Associates, Inc., and obtained by OSU from AFCRL.

The orbit model contained in the program written at OSU is a simple unperturbed Keplerian orbit. Our experiences with this program indicate that it can be used only with orbital arcs that do not exceed five or ten minutes in length. Even with adjustment of the initial conditions of the orbit, the errors introduced by insufficient modelling of the orbit reach about five to ten meters at the end of a five minute arc and ten to twenty meters at the end of a ten minute arc. This fact has necessitated that we split a pass into two separate segments in a few instances. Since data taken over arcs lasting 10 and even 20 or 30 minutes will be available from C-Band radars and lasers, we need a program whose orbit model accounts for at least the major perturbations of the orbit due to the gravity field.

The SAGA program contains an orbit model based on an expansion of the gravity field into spherical harmonics as far as (4, 4). This is totally adequate for short arc work, introducing modelling errors of less than a meter at the end of a 20 minute arc. The SAGA program also contains an extensive error model for both optical and range observations, as well as many other advanced features. On the other hand, the many advanced features make the SAGA program extremely slow running, so that it is extremely uneconomical to use for problems which do not require exercising an extensive error model.

It has become apparent to us that to perform our investigations economically we need a short arc adjustment program with an adequate orbit model and with a flexible error model, so that error model unknowns need not be solved for in problems in which they are not of interest. To achieve such a program, we have decided to combine the orbit and variational equation integration modules from the SAGA program with many modules from existing programs at OSU to produce a new short-arc mode adjustment program. The variable length error model from the old short arc programs is being retained, and most of the subroutines used for input/output processing, matrix manipulations, and time computations in the geometric mode programs will be used in the new short arc program. The orbit integrator from the SAGA program is extremely efficient, so that the running time of the new short arc program is not expected to be greatly increased over that of the old program.

Work on this task began during the last two weeks of the last reporting period. Because of the great number of subroutines that will be brought over unchanged from existing programs, the total effort required to put together and test the new program is not expected to exceed three months for one Research Assistant. The overall concept of the new program has been designed, and four new subroutines have been planned. The main features of the new program are as follows: The observations accepted are ranges from ground observing stations to the satellite with optical (camera) observations to be added later. The unknowns are the coordinates of the ground stations, the orbit elements for each pass, and a set of error model unknowns. The error model unknowns considered are a zero set term and a refraction term for each station on each pass. The error model is flexible, and each term may or may not be considered on any given pass. All error model terms are subject to a priori constraints. Options are included for a large number of possible weighted or absolute a priori constraints on, and between, the station coordinates.

The program is designed to be run on the IBM 360/75 of The Ohio State University. Because of the priority scheduling system used on this machine, it is desirable to limit the amount of core memory used by the program, by limiting the size of certain pertinent arrays. As presently designed the program is limited to a total of 40 ground stations, no more than 15 of which may observe on any given pass. The total number of error model unknowns which may be exercised on any given pass is limited to 10. Accumulation of the normal equations and partitioning of the normal equation matrix are used so that there is no limit on the number of observations in each pass or the total number of passes. These limits are designed to result in a program that can be run in the highest priority class. Any or all of these parameters may be easily increased by changing the dimensioning of the arrays, but this would result in a lower scheduling priority. On the other hand, these limitations are sufficient for almost all of the problems we expect to handle in the future.

A copy of a letter related to this programming effort is attached.

December 23, 1970

Mr. George Hadgigeorge
AFCRL
L. G. Hanscom Field
Bedford, Massachusetts 01730

Dear George:

I have recently been reading the copy of the SAGA program which you sent to us last spring. We are now interested in having a short arc program whose orbit model is sufficient to handle arcs of 20-30 minutes; although SAGA will do this, we find it to be too slow and expensive to be used frequently for a variety of problems. I have timed various modules of SAGA and find that a large amount of time is spent in computations relevant to the extensive error model. These computations are always performed, whether one wishes to fix the error model unknowns or not. We would like to have for ourselves a program in which the extensive error model is sacrificed in favor of efficiency. I am now trying to decide whether the most advisable approach will be for me to modify the coding of our copy of SAGA, or to use the orbit generating routines from SAGA in our own short arc program. At this time I favor the latter approach.

In this connection I have been studying the orbit generating routines in SAGA. Although I don't understand the exact version of Hartwell's equations that are being used, I have figured out how to generate an orbit and have performed several tests. I find the orbits routines to be excellent in their coding and surprisingly fast in their execution. Subroutine EXPAND, a long complicated subroutine which computes the coefficients of the time polynomials for the orbit position and velocity by Hartwell's recursive equations, is executed by our IBM 360/75 in 0.1 sec. Subroutine VARIEQ, another long complicated routine which computes the coefficients of the polynomials for the transition matrix (matrizant) terms, is also executed in 0.1 sec. This means that the orbit position and partial derivatives can be computed at several hundred points along a short arc with less than one second of computer time, which is quite phenomenal. I have also tested the accuracy of the integration by comparing the computed positions with those obtained using a predictorcorrector form of numerical integration and found the agreement to be quite excellent. Since the version of the routines you sent us uses only ten terms in the series, the series must be reinitialized every 400 seconds or so to maintain an accuracy of a few centimeters.

Although I have been able to use and test the orbit routines, there are many points I have not been able to resolve. I am listing several questions below; although I don't expect that you will be able to provide complete answers, I am hoping that you will pass them on to Jerry Trotter or someone else who can.

(1) Is there any place that I can find the version of Hartwell's recursive equations that are coded in subroutines EXPAND and VARIEQ? The articles by Hartwell that I have are (a) "Integration of Close Satellite Orbits and Solution of First Variational Equations by Recursive Analytic Continuation," presented at the Sixth Western National Meeting of the AGU, and (b) "The Recursive Formulation of a Taylor Series for Orbit Integration" an Appendix to a DBA report titled "Geodetic Data Analysis for GEOS-A, an Experimental Design." Neither of these give the form of the equations that were used in SAGA. I would like to see the equations so that I might understand better what is going on.

(2) Were the computations dealing with the coordinates of the center of mass thoroughly verified? I generated an orbit using non-zero values for the coordinates of the center of mass, and my tests indicated that neither the orbit position nor the partial derivatives were correct. I have seen the DBA report titled "A Theoretical Development for the Determination of the Center of Mass of the Earth from Artificial Satellite Observations," presented by Hartwell to the 1968 AGU meeting. It appears to me that the equations derived in this report are valid only for a non-rotating earth. However, the equation relating the partial derivatives of the orbit position with respect to the center of mass to the partial derivatives contained in the transition matrix is used in SAGA. Is there another reference justifying the use of these equations in the case of a rotating earth, where the time derivatives of the coordinates of the center of mass are not zero?

(3) Are any computations for atmospheric drag ever performed in SAGA?

I took out all of the coding at the end of EXPAND which computes the correction terms for drag, and I was not able to detect any difference in the computed positions and velocities.

Sincerely,

Charles R. Schwarz
Graduate Research Associate

CRS:eer

3. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time

Georges Blaha, Research Associate, part time

Charles R. Schwarz, Research Associate, part time

James P. Reilly, Research Associate, part time

Narendra K. Saxena, Research Associate, full time

Marvin C. Whiting, Research Assistant, part time

Evelyn E. Rist, Technical Assitant, part time

Deward R. Watts, Research Aide, part time

4. TRAVEL

Trips made by project personnel during the report period are:

Ivan I. Mueller
Flagstaff, Arizona, June 23 - July 4, 1970
To attend International Symposium on Mechanical Properties and Processes of the Mantle (partial support)

James P. Reilly
Wallops Island via Salisbury, Maryland, September 10-11, 1970
To attend technical discussions with Wolf Research Corporation and Wallops personnel

Marvin C. Whiting
Wallops Island via Salisbury, Maryland, September 10-11, 1970
To attend technical discussions with Wolf Research Corporation and Wallops personnel

Ivan I. Mueller
Greenbelt, Maryland, via Washington, D.C., September 16-18, 1970
To attend discussions at GSFC and NASA Headquarters

Narendra K. Saxena
Washington, D.C., October 12, 1970
To obtain data for project and meet with Mr. Meade, USCGS

Ivan I. Mueller
San Francisco, California, December 6-10, 1970
To attend the American Geophysical Union 1970 National Fall Meeting

5. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published under Grant No. NSR 36-008-003:

70 The Determination and Distribution of Precise Time
by Hans D. Preuss
April, 1966

71 Proposed Optical Network for the National Geodetic Satellite Program
by Ivan I. Mueller
May, 1966

82 Preprocessing Optical Satellite Observations
by Frank D. Hotter
April, 1967

86 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 1 of 3: Formulation of Equations
by Edward J. Krakiwsky and Allen J. Pope
September, 1967

87 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 2 of 3: Computer Programs
by Edward J. Krakiwsky, George Blaha, Jack M. Ferrier
August, 1968

88 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 3 of 3: Subroutines
by Edward J. Krakiwsky, Jack Ferrier, James P. Reilly
December, 1967

93 Data Analysis in Connection with the National Geodetic Satellite Program
by Ivan I. Mueller
November, 1967

OSU Department of Geodetic Science Reports published under Grant No. NGR 36-008-093:

100 Preprocessing Electronic Satellite Observations
by Joseph Gross
March, 1968

106 Comparison of Astrometric and Photogrammetric Plate Reduction Techniques for a Wild BC-4 Camera
by Daniel H. Hornbarger
March, 1968

110 Investigations into the Utilization of Passive Satellite Observational Data
by James P. Veach
June, 1968

114 Sequential Least Squares Adjustment of Satellite Triangulation and Trilateration in Combination with Terrestrial Data
by Edward J. Krakiwsky
October, 1968

118 The Use of Short Arc Orbital Constraints in the Adjustment of Geodetic Satellite Data
by Charles R. Schwarz
December, 1968

125 The North American Datum in View of GEOS I Observations
by Ivan I. Mueller, James P. Reilly, Charles R. Schwarz
June, 1969

139 Analysis of Latitude Observations for Crustal Movements
by M.G. Arur
June, 1970

140 SECOR Observations in the Pacific
by Ivan I. Mueller, James P. Reilly, Charles R. Schwarz, Georges Blaha
August, 1970

147 Gravity Field Refinement by Satellite to Satellite Doppler Tracking
by Charles R. Schwarz
December, 1970

The following papers were presented at various professional meetings:

"Report on OSU participation in the NGSP"
47th Annual meeting of the AGU, Washington, D. C., April 1966

"Preprocessing Optical Satellite Observational Data"
3rd Meeting of the Western European Satellite Subcommission, IAG, Venice, Italy, May 1967.

"Global Satellite Triangulation and Trilateration"
XIVth General Assembly of the IUGG, Lucerne, Switzerland, September 1967, (Bulletin Geodesique, March 1968).

"Investigations in Connection with the Geometric Analysis of Geodetic Satellite Data"
GEOS Program Review Meeting, Washington, D. C., Dec. 1967.

"Comparison of Photogrammetric and Astrometric Data Reduction Results for the Wild BC-4 Camera"
Conference on Photographic Astrometric Technique, Tampa, Fla., March 1968.

"Geodetic Utilization of Satellite Photography"
7th National Fall Meeting, AGU, San Francisco, Cal., Dec. 1968.

"Analyzing Passive-Satellite Photography for Geodetic Applications"
4th Meeting of the Western European Satellite Subcommission, IAG, Paris, Feb. 1969.

"Sequential Least Squares Adjustment of Satellite Trilateration"
50th Annual Meeting of the AGU, Washington, D. C., April 1969.

"The North American Datum in View of GEOS-I Observations"
8th National Fall Meeting of the AGU, San Francisco, Cal., Dec. 1969 and GEOS-2 Review Meeting, Greenbelt, Md., June 1970 (Bulletin Geodesique, June 1970).

"Experiments with SECOR Observations on GEOS-I"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"Experiments with Wild BC-4 Photographic Plates"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"Experiments with the Use of Orbital Constraints in the Case of Satellite Trails on Wild BC-4 Photographic Plates"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"GEOS-I SECOR Observations in the Pacific (Solution SP-7)"
National Fall Meeting of the American Geophysical Union, San Francisco, California, December 7-10, 1970.